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AN ECONOMIC MODEL OF THE LIFE-CYCLE RESEARCH PRODUCTIVITY OF SCIENTISTS

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Scientific productivity is constant as a scientist ages according to recent studies relying mainly on quantity measures of productivity. An economic model of the life-cycle productivity of scientists is presented which implies that the number of citations made to a scientist's previous work will decline with age. The implication could be consistent with the finding of constant quantity output with age if the decline in quality (as measured by number of citations per article) is large enough.

Lehman is a still oft-cited study (27 citations from 1979–1981), argued that the productivity of scientists declines continuously after age 30.[1] To defend his view, with regard to mathematicians, *Lehman* went through *Cajori's History of Mathematics*[2] in order to note all datable contributions to mathematics by then-deceased individuals. Of the 938 contributions on his initial list, 583 were made by mathematicians who lived to 70 or beyond. By matching the contributions to the mathematician's age when the contribution was made, *Lehman* came up with roughly the following profile: Although *Lehman* gives 16 causes for the decline in productivity after 30, (including increasing "maladjustment in sex life"), his first four emphasize declining physical capacities.[3]

Common wisdom still sides with *Lehman,* but the current consensus among the experts is that *Lehman* was wrong. Finding adumbration in the late 1950's papers of *Dennis,* the new view that productivity is constant with age is shared by *Stern, Zuckerman* and, most formidably, *Cole.*[4] Although *Cole* makes a flawed attempt to measure changes in quality of articles with age, his most solid evidence is that the quantity of articles published remains constant.[5] The evidence of *Stern* and *Zuckerman* also concerns quantity of publications and adds robustness to *Cole*'s result. As a policy implication of his results, *Cole* concludes that we need not fear, as we would if *Lehman* were right, that the increasing average age of scientists over the next 20 years will reduce the nation's "scientific capacity".[6]

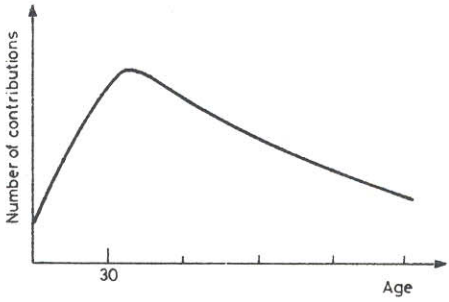

Fig. 1.

No less surprised than others by *Cole*'s findings are those economists whose human capital models imply that as the end of life approaches, the stock of human capital, and hence productivity, declines because no new investment offsets depreciation.[7] Since most scientific research takes place in the university and since the objective function of the university is not well understood, economists are divided about whether scientists' research is directly demanded by the university or is human capital that increases the scientists' productivity in other directly demanded activities such as teaching, fund-raising, or administration.[8] Whichever interpretation turns out to be most fruitful for other purposes, both have the same implication about the relation of research productivity to age: that research and productivity should increase up to some peak age and then decline.

A simple human capital investment model, following closely the work of *Becker*,[9] will be adapted here to the situation of the research scientist in order to derive testable implications concerning the scientist's life-cycle productivity. If science is an enterprise aimed at providing general explanations of an ever-wider range of phenomena, then economics has a plausible claim to being the most scientific of the 'social sciences'. The model presented here is characteristic of many economic models in that it explains behavior in terms of income maximization under constraints. To the extent that the behavior of scientists is explainable in such terms, we have further illustration of the power of the economist's model of man.[10] To the extent that the behavior of scientists is not explainable in such terms, then that will hearten those who view science as fundamentally different from other human activities in spirit and aim.[11]

The version of the model that follows could be reformulated to provide the greater rigor that is possible with the mathematically moe sophisticated techniques of the calculus of variations or of optimal control theory. Based on past applications of these techniques, few if any, new empirical implications would result. The techniques are

therefore avoided, in the belief that the added mathematical complexity would be too high a price to pay for the increase in rigor.

The model has the scientist maximizing V, the discounted sum of his income in the current year, i, and all future years until retirement, n. Income in any year is the product of time spent working, t_w, his stock of prestige capital, K, and the 'wage' rate per unit of human capital, W (assumed constant). The discount factor is $1/(1 + r)$ where the internal rate of return, r, is assumed constant across periods. The preceding, expressed formally is:

$$V = \sum_{i=1}^{n} \frac{WK_i t_{wi}}{(1 + r)^i} \tag{1}$$

For the research scientist, 'work' consists of administration, lecturing, dissertation supervising and 'gate-keeping' activities such as refereeing. The human capital that increases his value in these activities is taken to be his professional prestige which, in a more complete model, would itself be a function of the importance of a scientist's past work to research being done at the current frontiers. A scientist's professional prestige can thus he proxied by the current citations that his peers make to all of the scientist's past works. The scientist's total time in year i is labelled T_i and is allocated between time spent in academic work activities, t_{wi}, and time spent producing publications that will yield citations, t_{ki}. The time constraint is thus:

$$T_i = t_{wi} + t_{ki}. \tag{2}$$

The stock of prestige in year i, K_i, is given by the accounting identity:

$$K_i = K_{i-1} + g(t_{k_{i-2}}, K_{i-2}) + \delta K_{i-1} \tag{3}$$

where δ is the depreciation rate (assumed constant) of prestige and $g(t_{k_{i-2}}, K_{i-2})$ is the production function for prestige. (The motivation for the $i - 2$ subscript is that roughly two years pass before a publication begins to be cited.)

In this model there are two choice variables in each period. Since there are n periods, there are 2n choice variables over the entire span. Maximizing income (1) subject to constratints (2) and (3) is equivalent to maximizing the following *Lagrangean:*

$$L = \sum_{i=1}^{n} \frac{WK_i t_{wi}}{(1 + r)^i} + \mu_i(T - t_{wi} - t_{ki}) \tag{4}$$

where μ is the marginal value of time. Maximizing (4) results in the 2n equilibrium conditions:

$$\frac{\partial L}{\partial t_{wi}} = \frac{WK_i}{(1 + r)^i} - \mu_i = 0 \qquad i = 1, \ldots, n \tag{5}$$

$$\frac{\partial L}{\partial t_{ki}} = \sum_{j=1+2}^{n} \frac{Wt_{wj}}{(1+r)^j} \frac{\partial K_j}{\partial t_{ki}} - \mu_i = 0 \qquad i = 1, \ldots, n. \tag{6}$$

Since in equilibrium the marginal value of time, μ, must be equal in all its uses, (5) and (6) can be combined to yield:

$$0 = \frac{WK_i}{(1+r)^j} - \sum_{j=1+2}^{n} \frac{Wt_{wj}}{(1+r)^j} \frac{\partial K_j}{\partial t_{ki}} \qquad i = 1, \ldots, n. \tag{7}$$

Equation (7) shows that time spent producing publications, t_k, would decline for two reasons. The first is that the marginal return would decline as the years left diminished. The second assumes that the production function of human capital is non-neutral (i.e., that $\frac{\partial K_j}{\partial t_k}$ does not increase sufficiently with increasing K to offset the effect of increasing K on the value of time at work). In that case, time spent producing publications would decrease as the cost of investment, in income foregone, increased. Thus equation (7) implies the following age profiles for investment time, t_k, and prestige stock, K: The assumption of a constant 'wage' rate for each unit of prestige stock, implies that the life-cycle profile for income will have the same concave shape as the profile for K. On the standard assumption that added human capital in a period is a concave function of the time spent acquiring it in the period, the first profile implies the following profile for human capital invested (i.e., prestige):

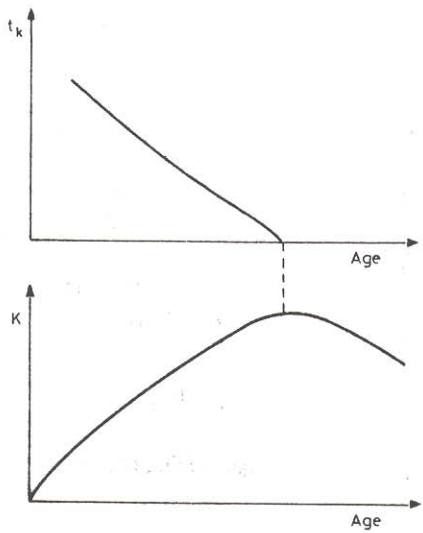

Fig. 2.

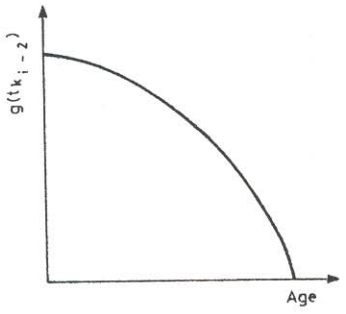

Fig. 3.

If a scientist's prestige is proxied by the current number of citations to his work, then the final profile implies that number of citations should decline with age. At first glance this result might seem inconsistent with the empirical findings of *Cole* and others about the constancy of quantity produced. Certainly it *does* contradict the generalization *Cole* makes from his findings that the rate of scientific advance need not be slowed by an aging of the scientific population. The specific findings of *Cole*, however, might be consistent with the model if scientific productivity had two components: quantity and quality. Then, one component (quantity) might remain nearly constant over most of the life-cycle provided the other (quality) declined sufficiently to reduce to over-all investment in prestige.

To see somewhat more rigorously how this might happen, consider the simple case:

$$g_i = N_i Q_i \tag{8}$$

where g_i is new citations in period i, N_i is number of articles in period i and Q_i is average citations per article in period i. If N and Q were each simple exponential functions of time in period i (henceforth the i subscripts will be supressed) then:

$$N = t_n^a \tag{9}$$

$$Q = t_q^b \tag{10}$$

where t_n and t_q equal t_k, the total time spent investing:

$$t_k = t_n + t_q \tag{11}$$

If t_k can be taken as determined by an earlier stage of the maximization process, then the scientist's problem at this stage may be interpreted as how to maximize his prestige

(NQ) subject to the time constraint (11). Solving for the factor demand equations results in:

$$t_n^* = \frac{at_k}{a+b} \tag{12}$$

$$t_q^* = \frac{bt_k}{a+b} \tag{13}$$

Substituting (12) into (9) and (13) into (10) yields:

$$N = \left(\frac{at_k}{a+b}\right)^a \tag{14}$$

$$Q = \left(\frac{bt_k}{a+b}\right)^b \tag{15}$$

If t_k decreased over the life cycle then from (14) and (15) the product NQ must decrease. If, as *Cole* et al. believe, N is constant over the life cycle, then Q must diminish and the ratio $\frac{N}{Q}$ must increase. To see what this implies about the relative time intensity of production of quantity and quality express the ratio as:

$$\frac{N}{Q} = \frac{\left(\frac{at_k}{a+b}\right)^a}{\left(\frac{bt_k}{a+b}\right)^b} \tag{16}$$

and then differentiate by $-t_k$:

$$\frac{d\frac{N}{Q}}{d-t_k} \gtrless 0 \quad \text{as} \quad a \gtrless b \tag{17}$$

So as the time spent investing decreases, an increase in the ratio $\frac{N}{Q}$ implies that $a < b$, which means that quality is more time intensive than quantity. The important result is that if quality is more time-intensive than quantity, then an overall decline in research productivity with age is consistent with the constant quantity output found by *Cole*, provided that quality declines sufficiently with age.

The simple economic model of citations presented here has provided plausible and testable implications about the life-cycle profile of citations to a scientist's work. Most notably, the model implies that: (1) the life-cycle salary profile is concave, (2) the life cycle citation profile is concave and (3) either the quantity of output or the average quality per article

(or both) decline with age. The model thus has important bearing on the controversial relationship between age and scientific productivity. One avenue for future research is to test the model using longitudinal data on research scientists. A new data set containing information on citations, age and salaries for scientists and mathematicians at Berkeley and the University of Illinois at Urbana is now being analyzed in order to test both the implications of the model and the robustness of previous empirical generalizations. Another possible avenue for future research would be to investigate the relative time intensity of various scientific activities in order to predict more precisely how participation in these activities will change with age.

Notes and References

[1] H. LEHMAN, *Age and Achievement*, Oxford University Press, London, 1953, pp. 3–21 and 308–323 (on mathematicians see especially pp. 8 and 311).

[2] F. CAJORI, *A History of Mathematics*, 2nd ed., the Macmillan Co., New York 1922.

[3] LEHMAN, pp. 328–329.

[4] W. DENNIS, Age and Achievement: A Critique, *Journal of Gerontology*, 11, No. 3, (July 1956) 331–333; Age and Productivity among Scientists, *Science*, 123 (April 27, 1956) 724–725; The Age Decrement in Outstanding Scientific Contributions: Fact or Artifact?, *The American Psychologist*, 13, (Aug. 1958) 457–460; Creative Productivity between the Ages of 20 and 80 Years, *Journal of Gerontology*, 21, No. 1, (Jan. 1966) 1–8; N. STERN, Age and Achievement in Mathematics: A Case-Study in the Sociology of Science, *Social Studies of Science*, 8 (1978) 127–140; H. ZUCKERMAN, Appendix E: Age-Specific Annual Rates of Productivity of Laureates and Matched Sample of Scientists Who Survived to Each Age, in *Scientific Elite*, The Free Press, New York 1977, pp. 301–302; S. COLE, Age and Scientific Performance, *American Journal of Sociology*, 84 No. 4, (Jan. 1979) 958–977; I originally thought that a third current position, distinct from those of *Lehman* and *Cole*, was *Price*'s version of the 'contagion' model (D. de SOLLA PRICE, A General Theory of Bibliometric and Other Cumulative Advantage Processes, *Journal of the American Society for Information Science* (Sept.–Oct. (1976) 292–306.) I interpreted *Price*'s model as implying that for those who 'caught on' productivity would increase with age (see pp. 303–304 of above). But *Price* claims that I misinterpreted him and that in fact he agrees with *Cole:* "My own feeling is that productivity in quantity and quality are approximately constant with age though of course productivity must rise from zero to its level during the first few years of scholarly activity and towards the end of life there may well be, in some but not all cases, a tendency to make a slight decline." (letter, June 23, 1980). See also: D. de SOLLA PRICE, The Productivity of Research Scientists, in *Yearbook of Science and the Future 1975, Encyclopedia Britannica*, 1974. An application of *Price*'s contagion model that, from the title, would seem to bear on the age/productivity issue is: Y. WEISS, L. LILLARD, Output Variability, Academic Labor Contracts and Waiting Time for Promotion, in R. G. EHRENBERG, Ed., *Research in Labor Economics* 5, (1982) 157–188. *Weiss* and *Lillard*, however, want to insulate their model from any prima facie implications it might have for the issue: "For the purpose of our analysis it is *not* necessary to assume that output is measurable. One can think of abstract and *unobserved* 'promotion capital' which is related, but not necessarily identical to the number of published papers. One can maintain the specific distribution implications

of the model for time to promotion without necessarily maintaining the specific distributional implications for papers production" (p. 172, italics in original). Despite *Price*'s concurrence with *Cole* et al., the current consensus about age/productivity profiles is not total. For a sample of 20 past presidents of the American Economic Association, *Eagly* found that number of articles peaked at 6 per year at age 47 and declined to about 2 per year at age 67 (R. EAGLY, Contemporary Profile of Conventional Economists, *History of Political Economy*, 6, No. 1, (1974) 88). For a sample of 5079 scientists, *Bayer* and *Dutton* compare 6 functional forms of the quantity/age relation of find out which one provides the 'best fit'. Since they require that all the betas be significant for the model that 'fits best', they rule out from the start the possibility that productivity is constant with age (in which case, the constant, but none of the betas would be significant). Of the remaining 6 functional forms, they find that a 4th degree polynomial of age (which they call the spurt-obsolescence function) provides the best fit (A. E. BAYER, J. E. DUTTON, Career Age and Research-Professional Activities of Academic Scientists, *Journal of Higher Education*, 48, No. 3. (May/June, 1977) pp. 259–282).

[5] Noting correctly that "longitudinal data are superior to cross-sectional data in measuring the effects of age on scientific productivity," (p. 964) *Cole* uses longitudinal data for a sample of 497 mathematicians who received their Ph. D.'s between 1947 and 1950. For five year periods between 1950 and 1975, *Cole* finds that mean citations (in 1975) to articles written in each period varied only slightly around one citation per article (p. 695). From this, *Cole* concludes that quality of output does not decline with age. *Cole*'s method can be criticized on several grounds. Perhaps the most important of these is that in the early periods the sample includes work by low-quality mathematicians who later stop publishing. Since their work diminishes the group mean in the early periods and since the group mean remains constant over all periods, it follows that the mean quality of work for those mathematicians who continue to publish must decline with age.

[6] COLE, p. 959.

[7] Among the major works in the optimal life-cycle investment literature are G. S. BECKER, *Human Capital*, 2nd ed., Columbia University Press New York, 1975; Y. BEN-PORATH, The Production of Human Capital and the Life Cycle of Earnings, *Journal of Political Economy*, 75, No. 4, Pt. I (Aug. 1967) 352–365; G. R. GHEZ, G. S. BECKER, *The Allocation of Time and Goods over the Life Cycle*, Columbia University Press, New York, 1975; J. J. HECKMAN, A Life-Cycle Model of Earnings, Learning, and Consumption, *Journal of Political Economy*, 84, No. 4, Pt. 2 (Aug. 1967) S11–S44; H. E. RYDER, F. STAFFORD, P. E. STEPHAN, Labor, Leisure and Training Over the Life Cycle, *International Economic Review*, 17, No. 3 (Oct. 1976) 651–674; S. ROSEN, A Theory of Life Earnings, *Journal of Political Economy*, 84, No. 4, Pt. 2 (Aug. 1976) S45–S67; and A. S. BLINDER, Y. WEISS, Human Capital and Labor Supply: A Synthesis, *Journal of Political Economy*, 84, No. 3 (June 1976) 449–472.

[8] *Weiss* and *Lillard* treat publications as "promotion capital" although their model puts more emphasis on stochastic contagion processes than does the typical optimization-under-constraints human capital model. (See Y. WEISS, L. LILLARD, Output Variability, Academic Labor Contracts and Waiting Time for Promotion, in: R. G. EHRENBERG, Ed., *Research in Labor Economics*, 5, (1982) 157–188).

[9] G. S. BECKER, *Human Capital*, esp. pp. 63–67.

[10] One of the most articulate recent defenses of the power of the economic model of man is: G. J. STIGLER, G. S. BECKER, De Gustibus Non Est Disputandum, *American Economic Review* 67 (March 1977) 76–90.

[11] For examples see: A. SMITH, *The Theory of Moral Sentiments*, Oxford University Press Oxford 1976, p. 126; and an unnamed biochemist quoted in: W. O. HAGSTROM, *The Scientific Community*, Basic Books, New York, Inc. 1965, p. 125.